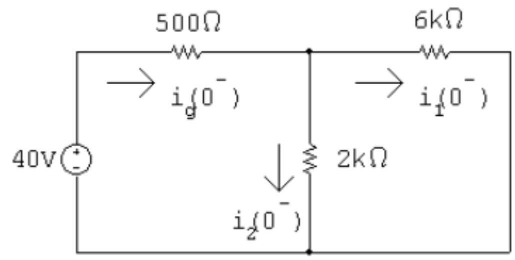
EE 215 Fundamentals of Electrical Engineering Spring 2010
Problem #7 Solution

P 7.4 [a] $t < 0$



$2\,\text{k}\Omega \| 6\,\text{k}\Omega = 1.5\,\text{k}\Omega$

Find the current from the voltage source by combining the resistors in series and parallel and using Ohm's law:

$$i_g(0^-) = \frac{40}{(1500 + 500)} = 20\,\text{mA}$$

Find the branch currents using current division:

$$i_1(0^-) = \frac{2000}{8000}(0.02) = 5\,\text{mA}$$

$$i_2(0^-) = \frac{6000}{8000}(0.02) = 15\,\text{mA}$$

[b] The current in an inductor is continuous. Therefore,

$$i_1(0^+) = i_1(0^-) = 5\,\text{mA}$$

$$i_2(0^+) = -i_1(0^+) = -5\,\text{mA} \qquad \text{(when switch is open)}$$

[c] $\tau = \dfrac{L}{R} = \dfrac{0.4 \times 10^{-3}}{8 \times 10^3} = 5 \times 10^{-5}\,\text{s}; \qquad \dfrac{1}{\tau} = 20{,}000$

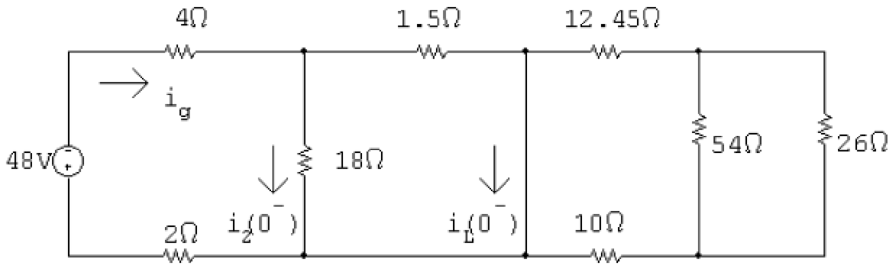
$$i_1(t) = i_1(0^+)e^{-t/\tau} = 5e^{-20{,}000t}\,\text{mA}, \qquad t \geq 0$$

[d] $i_2(t) = -i_1(t) \qquad \text{when} \quad t \geq 0^+$

$$\therefore \quad i_2(t) = -5e^{-20{,}000t}\,\text{mA}, \qquad t \geq 0^+$$

[e] The current in a resistor can change instantaneously. The switching operation forces $i_2(0^-)$ to equal $15\,\text{mA}$ and $i_2(0^+) = -5\,\text{mA}$.

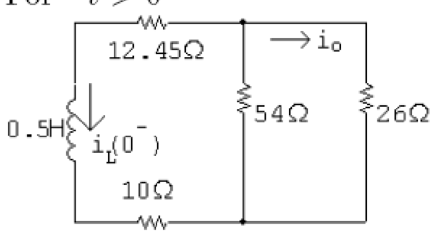
P 7.6 For $t < 0$



$$i_g = \frac{-48}{6 + (18\|1.5)} = -6.5\,\mathrm{A}$$

$$i_L(0^-) = \frac{18}{18 + 1.5}(-6.5) = -6\,\mathrm{A} = i_L(0^+)$$

For $t > 0$



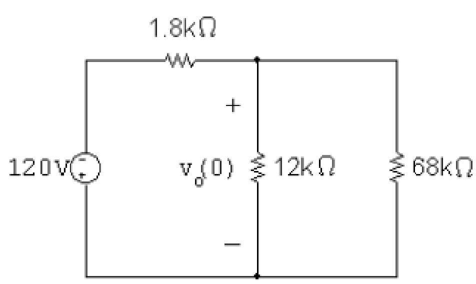
$$i_L(t) = i_L(0^+)e^{-t/\tau}\,\mathrm{A}, \qquad t \geq 0$$

$$\tau = \frac{L}{R} = \frac{0.5}{10 + 12.45 + (54\|26)} = 0.0125\,\mathrm{s}; \qquad \frac{1}{\tau} = 80$$

$$i_L(t) = -6e^{-80t}\,\mathrm{A}, \qquad t \geq 0$$

$$i_o(t) = \frac{54}{80}(-i_L(t)) = \frac{54}{80}(6e^{-80t}) = 4.05e^{-80t}\,\mathrm{V}, \qquad t \geq 0^+$$

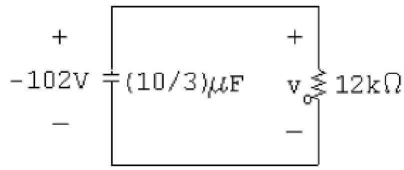
P 7.25 [a] $t < 0$:



$$R_{\mathrm{eq}} = 12\,\mathrm{k}\|8\,\mathrm{k} = 10.2\,\mathrm{k}\Omega$$

$$v_o(0) = \frac{10{,}200}{10{,}200 + 1800}(-120) = -102\,\text{V}$$

$t > 0$:



$$\tau = [(10/3) \times 10^{-6})(12{,}000) = 40\,\text{ms}; \qquad \frac{1}{\tau} = 25$$

$$v_o = -102e^{-25t}\,\text{V}, \quad t \geq 0$$

$$p = \frac{v_o^2}{12{,}000} = 867 \times 10^{-3}e^{-50t}\,\text{W}$$

$$w_{\text{diss}} = \int_0^{12\times 10^{-3}} 867 \times 10^{-3}e^{-50t}\,dt$$
$$= 17.34 \times 10^{-3}(1 - e^{-50(12\times 10^{-3})}) = 7824\,\mu\text{J}$$

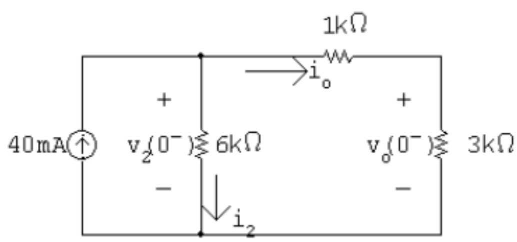
[b] $w(0) = \left(\frac{1}{2}\right)\left(\frac{10}{3}\right)(102)^2 \times 10^{-6} = 17.34\,\text{mJ}$

$$0.75w(0) = 13\,\text{mJ}$$

$$\int_0^{t_o} 867 \times 10^{-3}e^{-50x}\,dx = 13 \times 10^{-3}$$

$$\therefore \quad 1 - e^{-50t_o} = 0.75; \qquad e^{50t_o} = 4; \quad \text{so} \quad t_o = 27.73\,\text{ms}$$

P 7.26 [a] $t < 0$:



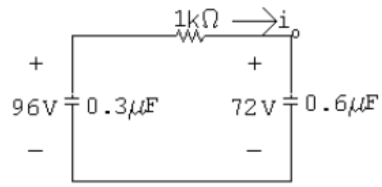
$$i_o(0^-) = \frac{6000}{6000 + 4000}(40\,\text{m}) = 24\,\text{mA}$$

$$v_o(0^-) = (3000)(24\,\text{m}) = 72\,\text{V}$$

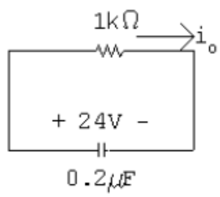
$$i_2(0^-) = 40 - 24 = 16\,\text{mA}$$

$$v_2(0^-) = (6000)(16\,\text{m}) = 96\,\text{V}$$

$t > 0$

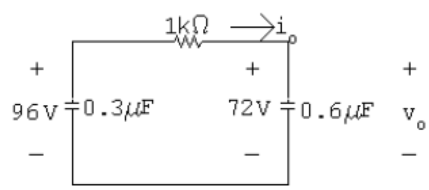


$$\tau = RC = (1000)(0.2 \times 10^{-6}) = 200\,\mu\text{s}; \qquad \frac{1}{\tau} = 5000$$



$$i_o(t) = \frac{24}{1 \times 10^3} e^{-t/\tau} = 24e^{-5000t}\,\text{mA}, \qquad t \geq 0^+$$

[b]



$$\begin{aligned} v_o &= \frac{1}{0.6 \times 10^{-6}} \int_0^t 24 \times 10^{-3} e^{-5000x}\,dx + 72 \\ &= (40{,}000) \frac{e^{-5000x}}{-5000} \Big|_0^t + 72 \\ &= -8e^{-5000t} + 8 + 72 \\ v_o &= [-8e^{-5000t} + 80]\,\text{V}, \qquad t \geq 0 \end{aligned}$$

[c] $w_{\text{trapped}} = (1/2)(0.3 \times 10^{-6})(80)^2 + (1/2)(0.6 \times 10^{-6})(80)^2$

$w_{\text{trapped}} = 2880\,\mu\text{J}.$

Check:

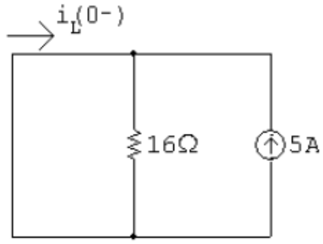
$$w_{\text{diss}} = \frac{1}{2}(0.2 \times 10^{-6})(24)^2 = 57.6\,\mu\text{J}$$

$$w(0) = \frac{1}{2}(0.3 \times 10^{-6})(96)^2 + \frac{1}{2}(0.6 \times 10^{-6})(72)^2 = 2937.6\,\mu\text{J}.$$

$$w_{\text{trapped}} + w_{\text{diss}} = w(0)$$

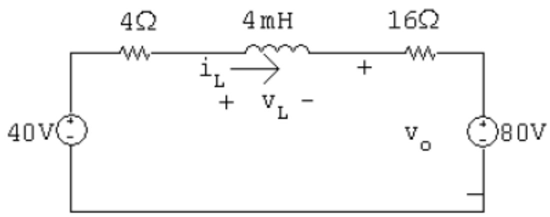
$2880 + 57.6 = 2937.6 \qquad$ OK.

P 7.35 [a] $t < 0$



$$i_L(0^-) = -5\,\text{A}$$

$t > 0$



$$i_L(\infty) = \frac{40 - 80}{4 + 16} = -2\,\text{A}$$

$$\tau = \frac{L}{R} = \frac{4 \times 10^{-3}}{4 + 16} = 200\,\mu\text{s}; \qquad \frac{1}{\tau} = 5000$$

$$i_L = i_L(\infty) + [i_L(0^+) - i_L(\infty)]e^{-t/\tau}$$

$$= -2 + (-5 + 2)e^{-5000t} = -2 - 3e^{-5000t}\,\text{A}, \qquad t \geq 0$$

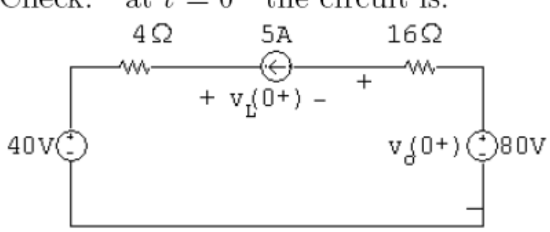
$$v_o = 16i_L + 80 = 16(-2 - 3e^{-5000t}) + 80 = 48 - 48e^{-5000t}\,\text{V}, \qquad t \geq 0$$

[b] $v_L = L\frac{di_L}{dt} = 4 \times 10^{-3}(-5000)[-3e^{-5000t}] = 60e^{-5000t}\,\text{V}, \qquad t \geq 0^+$

$$v_L(0^+) = 60\,\text{V}$$

From part (a) $\qquad v_o(0^+) = 0\,\text{V}$

Check: at $t = 0^+$ the circuit is:



$$v_L(0^+) = 40 + (5\,\text{A})(4\,\Omega) = 60\,\text{V}, \qquad v_o(0^+) = 80 - (16\,\Omega)(5\,\text{A}) = 0\,\text{V}$$

P 7.56 [a] Use voltage division to find the initial value of the voltage:

$$v_c(0^+) = v_{9k} = \frac{9\,\mathrm{k}}{9\,\mathrm{k} + 3\,\mathrm{k}}(120) = 90\,\mathrm{V}$$

[b] Use Ohm's law to find the final value of voltage:

$$v_c(\infty) = v_{40k} = -(1.5 \times 10^{-3})(40 \times 10^3) = -60\,\mathrm{V}$$

[c] Find the Thévenin equivalent with respect to the terminals of the capacitor:

$$V_{\mathrm{Th}} = -60\,\mathrm{V}, \qquad R_{\mathrm{Th}} = 10\,\mathrm{k} + 40\,\mathrm{k} = 50\,\mathrm{k}\Omega$$

$$\tau = R_{\mathrm{Th}}C = 1\,\mathrm{ms} \ = 1000\,\mu\mathrm{s}$$

[d] $v_c = v_c(\infty) + [v_c(0^+) - v_c(\infty)]e^{-t/\tau}$

$$= -60 + (90 + 60)e^{-1000t} = -60 + 150e^{-1000t}\,\mathrm{V}, \quad t \geq 0$$

We want $v_c = -60 + 150e^{-1000t} = 0$:

Therefore $\quad t = \dfrac{\ln(150/60)}{1000} = 916.3\,\mu\mathrm{s}$

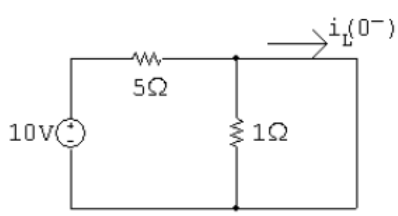
$$v_o(0^-) = v_o(0^+) = 40\,\mathrm{V}$$

$$v_o(\infty) = 80\,\mathrm{V}$$

$$\tau = (0.16 \times 10^{-6})(6.25 \times 10^3) = 1\,\mathrm{ms}; \qquad 1/\tau = 1000$$

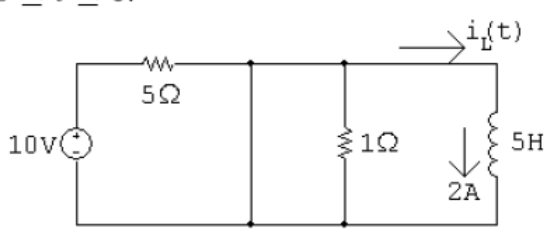
$$v_o = 80 - 40e^{-1000t}\,\mathrm{V}, \qquad t \geq 0$$

P 7.70 $t < 0$:



$$i_L(0^-) = 10\,\mathrm{V}/5\,\Omega = 2\,\mathrm{A} = i_L(0^+)$$

$0 \leq t \leq 5$:

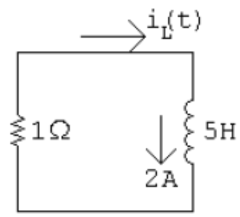


$\tau = 5/0 = \infty$

$i_L(t) = 2e^{-t/\infty} = 2e^{-0} = 2$

$i_L(t) = 2\,\mathrm{A} \quad 0 \leq t \leq 5\,\mathrm{s}$

$5 \leq t < \infty$:



$$\tau = \frac{5}{1} = 5\,\mathrm{s}; \qquad 1/\tau = 0.2$$

$$i_L(t) = 2e^{-0.2(t-5)}\,\mathrm{A}, \quad t \geq 5\,\mathrm{s}$$

P 8.2 [a] $i_{\mathrm{R}}(0) = \dfrac{15}{200} = 75\,\mathrm{mA}$

$$i_{\mathrm{L}}(0) = -45\,\mathrm{mA}$$

$$i_{\mathrm{C}}(0) = -i_{\mathrm{L}}(0) - i_{\mathrm{R}}(0) = 45 - 75 = -30\,\mathrm{mA}$$

[b] $\alpha = \dfrac{1}{2RC} = \dfrac{1}{2(200)(0.2 \times 10^{-6})} = 12{,}500$

$$\omega_o^2 = \frac{1}{LC} = \frac{1}{(50 \times 10^{-3})(0.2 \times 10^{-6})} = 10^8$$

$$s_{1,2} = -12{,}500 \pm \sqrt{1.5625 \times 10^8 - 10^8} = -12{,}500 \pm 7500$$

$$s_1 = -5000 \text{ rad/s}; \qquad s_2 = -20{,}000 \text{ rad/s}$$

$$v = A_1 e^{-5000t} + A_2 e^{-20{,}000t}$$

$$v(0) = A_1 + A_2 = 15$$

$$\frac{dv}{dt}(0) = -5000A_1 - 20{,}000A_2 = \frac{-30 \times 10^{-3}}{0.2 \times 10^{-6}} = -15 \times 10^4 \mathrm{V/s}$$

Solving, $A_1 = 10$; $A_2 = 5$

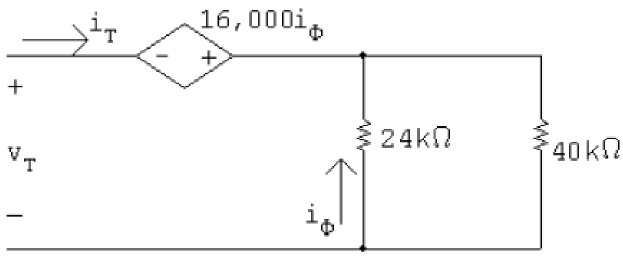
$$v = 10e^{-5000t} + 5e^{-20{,}000t}\,\mathrm{V}, \qquad t \geq 0$$

$$
\begin{aligned}
\text{[c]} \quad i_C &= C\frac{dv}{dt} \\
&= 0.2\times 10^{-6}[-50{,}000e^{-5000t} - 100{,}000e^{-20{,}000t}] \\
&= -10e^{-5000t} - 20e^{-20{,}000t}\,\text{mA}
\end{aligned}
$$

$$i_R = 50e^{-5000t} + 25e^{-20{,}000t}\,\text{mA}$$

$$i_L = -i_C - i_R = -40e^{-5000t} - 5e^{-20{,}000t}\,\text{mA}, \quad t \geq 0$$

P 8.21



$$v_T = -16{,}000 i_\phi + i_T(15{,}000) = -16{,}000\frac{-i_T(40)}{64} + i_t(15{,}000)$$

$$\frac{v_T}{i_T} = 10{,}000 + 15{,}000 = 25\,\text{k}\Omega$$

$$V_o = \frac{4000}{5000}(7.5) = 6\,\text{V}; \qquad I_o = 0$$

$$i_C(0) = -i_R(0) - i_L(0) = -\frac{6}{25{,}000} = -240\,\mu\text{A}$$

$$\frac{i_C(0)}{C} = \frac{-240\times 10^{-6}}{4\times 10^{-9}} = -60{,}000$$

$$\omega_o^2 = \frac{1}{LC} = \frac{10^9}{(4)(15.625)} = 16\times 10^6; \qquad \omega_o = 4000\ \text{rad/s}$$

$$\alpha = \frac{1}{2RC} = \frac{10^9}{(2)(4)(25\times 10^3)} = 5000\ \text{rad/s}$$

$\alpha^2 > \omega_0^2$ so the response is overdamped

$$s_{1,2} = -5000 \pm \sqrt{5000^2 - 4000^2} = -5000 \pm 3000\ \text{rad/s}$$

$$v_o = A_1 e^{-2000t} + A_2 e^{-8000t}$$

$$v_o(0) = A_1 + A_2 = 6\,\text{V}$$

$$\frac{dv_o}{dt}(0) = -2000A_1 - 8000A_2 = -60{,}000$$

$$\therefore \quad A_1 = -2\,\text{V}; \qquad\qquad A_2 = 8\,\text{V}$$

$$v_o = 8e^{-8000t} - 2e^{-2000t}\,\text{V}, \qquad t \geq 0$$